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SY-R2-71

ANALYSIS OF RISK
FOR THE MATERIEL ACQUISITION PROCESS
PART II: UTILITY THEORY

U.S. ARMY WEAPONS COMMAND

BY

JOHN D. HWANG

SEE AD 715394

SYSTEMS ANALYSIS DIRECTORATE
HEADQUARTERS, U.S. ARMY WEAPONS COMMAND
ROCK ISLAND, ILLINOIS

MAY 1971

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13. ABSTRACT

This is the second paper in the series devoted to the subject of analysis of risk for the materiel acquisition process. It is emphasized that risk analysis must interface with decision analysis to facilitate decision-making for major developmental programs. A concise discussion of utility theory, lotteries, and techniques to elicit utility functions is presented, as well as a set of utility axioms. The concepts are used for the decision analysis of a hypothetical example.

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Analysis of risk						
Materiel acquisition process						
Utility theory						
Decision analysis						

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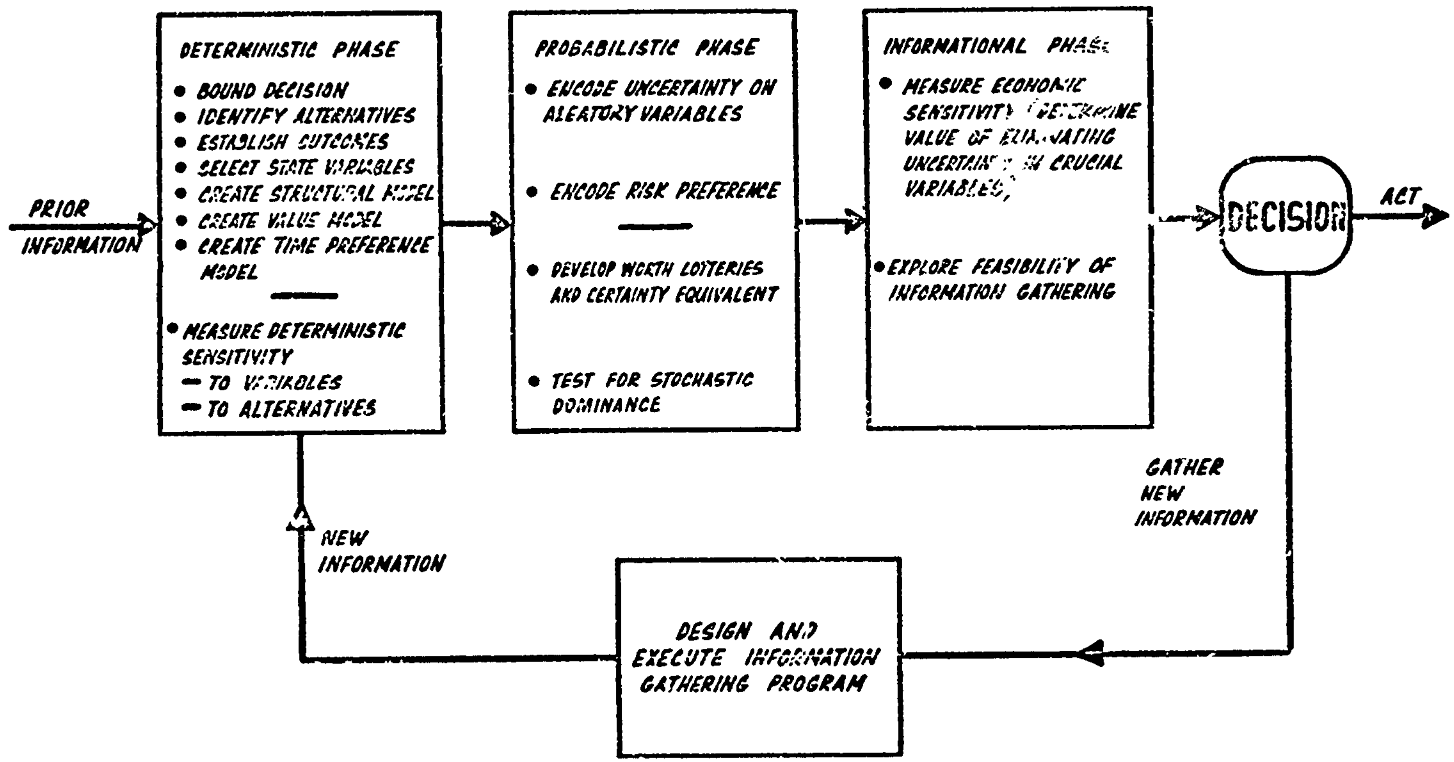
I. INTRODUCTION

This is the second paper in the series devoted to the subject of analysis of risk for the materiel acquisition process. In the first paper (Hwang, 1970), the analysis of risk was structured to show that it has close affinity to systems analysis and adds a new dimension, in terms of a probability measure, to integrate the three dimensions of life-cycle cost, time to complete acquisition, and performance of a program in the materiel acquisition process. Secondly, numerous applicable techniques of statistical decision theory were presented, including decision tree analysis and subjective judgment collection. Thirdly, methods for risk analysis of the concept formulation and contract definition phases of the acquisition cycle were exhibited.

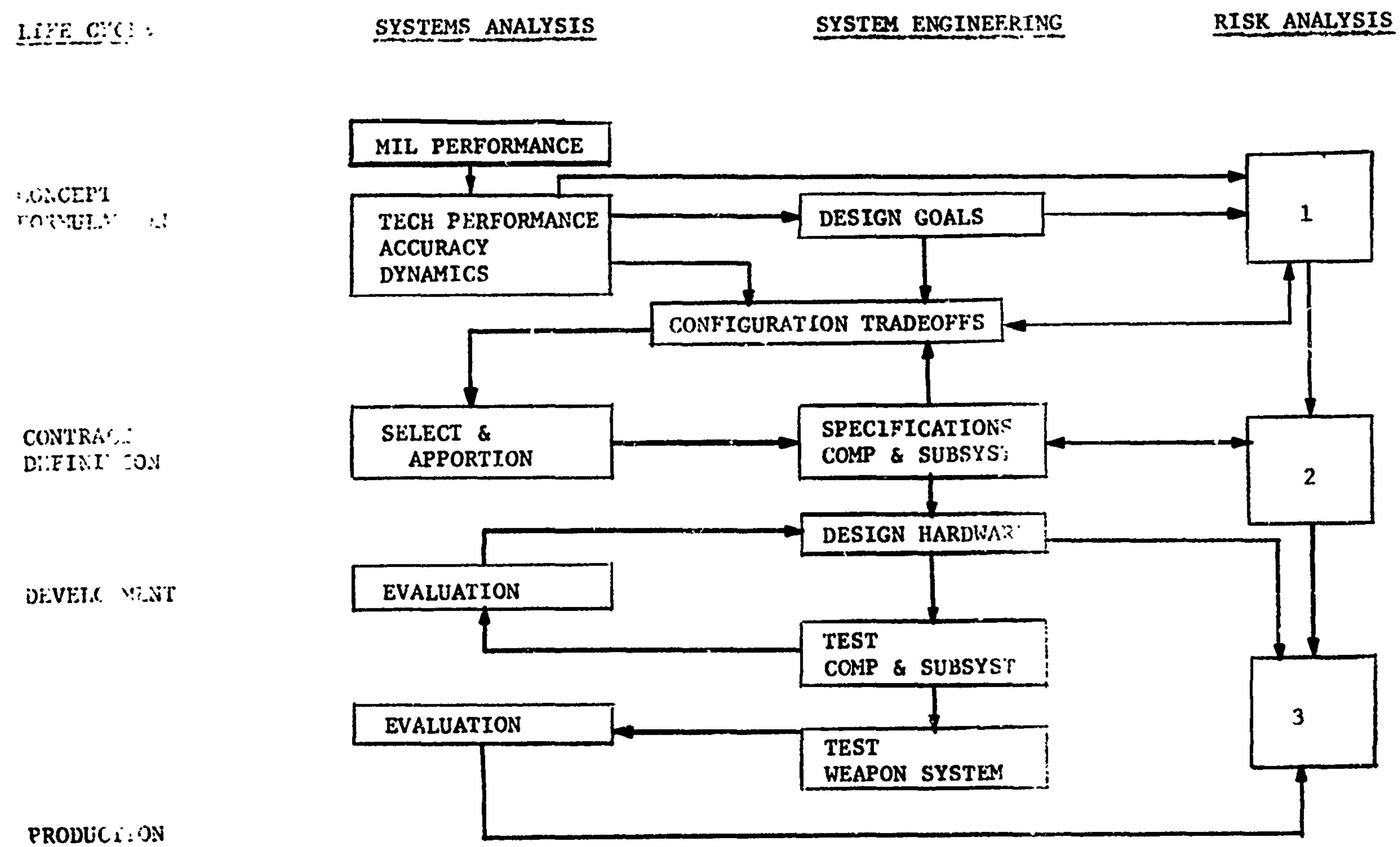
Risk analysis has been defined as the disciplined process, essential to program decision making, involving the application of broad class of qualitative and quantitative techniques for analyzing, quantifying, and reducing the uncertainties associated with the realization of cost, time, and performance goals of large-scale military projects. Decision analysis has arisen which merges statistical decision theory with systems analysis (Howard, 1968); this is a logical procedure for balancing the factors that influence a decision. If decision analysis is properly tied to risk analysis, then we have a truly balanced appraisal of the project. To emphasize again the interfacing between risk analysis and decision analysis, we observe that much information is generated throughout the materiel acquisition process. Particularly in the early phases of the process, analyses and software techniques yield information in terms of the most decisive elements consisting of cost, time, and performance in some integrated measures such as cost-effectiveness indices. It is seen that although the studies show the materiel is cost-effective over a specified time-frame, there is usually very little quantitative information as to the assurance that the program would be successful, even given the specified time and allotted the estimated cost resources needed for the pre-established performance level. It is precisely this missing link that is vital to program success. Analysis of risk is designed to fill this gap.

Stanford Research Institute has designed a generalized model to describe the decision analysis cycle as shown on the following page (Matheson, 1969). The basic elements in decision analysis are summarized in three phases: deterministic, probabilistic, and information. Interested readers can consult Spetzler (1968) and North (1968) for more detailed discussions and examples.

Risk analysis is by nature an iterative process and must be up-dated and validated at regular intervals. It has been proposed that risk analysis be carried out at least three times in the acquisition process: during concept formulation, during contract definitions, and prior to a production decision. These analyses should be coordinated with key decision points of the acquisition cycle. On Page 4 is a diagram showing the interface among systems analysis, system engineering, and risk analysis, relative to the materiel acquisition process.



THE DECISION ANALYSIS CYCLE


LIFE CYCLE
SYSTEMS ANALYSIS
SYSTEM ENGINEERING
RISK ANALYSIS
CONCEPT
CONTRACT
DEFINITION
PRODUCTION
MIL PERFORMANCE
TECH PERFORMANCE
ACCURACY
DYNAMICS
DESIGN GOALS
1
CONFIGURATION TRADEOFFS
SELECT &
APPORTION
SPECIFICATIONS
COMP & SUBSYS
2
DESIGN HARDWARE
EVALUATION
TEST
COMP & SUBSYST
3
EVALUATION
TEST
WEAPON SYSTEM

In the first paper, one notable missing area is the utility theory reflecting uncertainties, values, and preferences relevent to a decision. The purpose of this paper is to provide a concise discussion of utility theory, lotteries, and techniques to elicit utility functions. The concepts are used for the decision analysis of a hypothetical example. In particular, cost-sensitivity analysis (Petruschell, 1968) has been defined as the process of determining how variations in the specifications of a particular system, either in design or operation, affect the requirements of that system for resources. The example illustrates the cost uncertainty, thereby establishing the risks in the decisions.

II. UTILITY THEORY AND LOTTERY

Utility is a term interpreted in many different ways at the present time. In order to establish a basis for subsequent discussions, a simple but mathematically concise set of utility axioms is presented in the Appendix (Luce and Raiffa, 1957). This set is one version of utility theory originally created by von Neumann and Morganstern (1953); another version is found in Ferguson (1967).

Consider three alternatives, h_1, h_2, and h_3, such that

$$h_1 \gtrsim h_2 \gtrsim h_3 ,$$

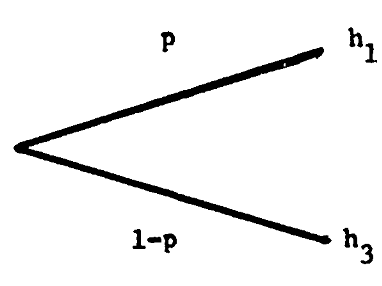
where " $\gtrsim$ " denotes the relation "at least as preferred as". A lottery is a chance mechanism which yields the outcome alternatives with the known probabilities of occurrences. We denote

$$(ph_1, (1-p)\ h_3)$$

to mean a lottery with outcomes h_1 and h_3 with respective probabilities of occurrence p and $(1-p)$; in diagramatic form,

$$(ph_1, (1-p)\ h_3)$$

is equivalent to

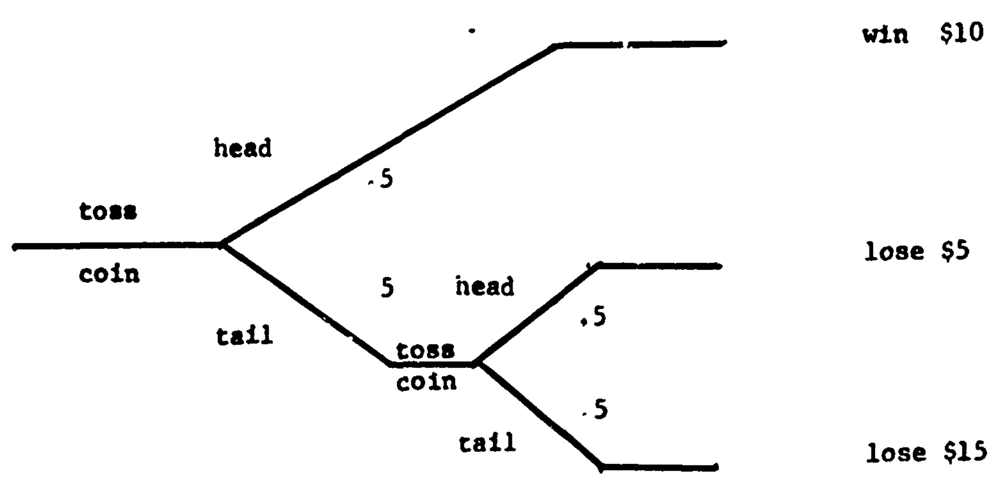


Intuitively, Luce and Raiffa describe the nature of utility as follows:

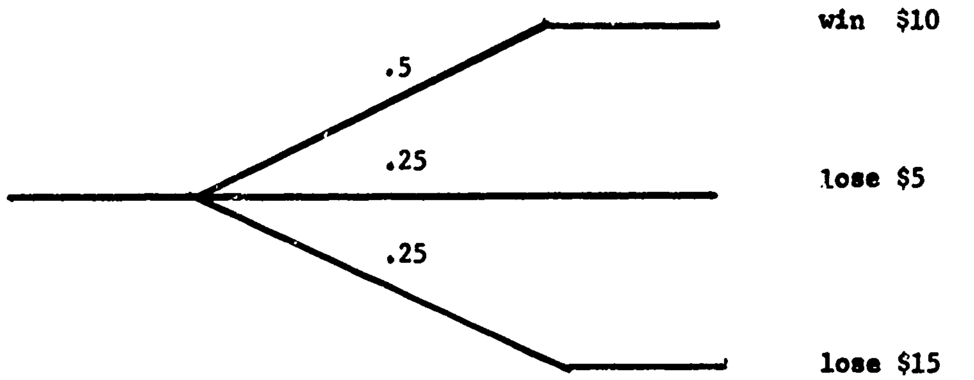
a. Comparability - Any two alternatives shall be comparable; i. e., one alternative is preferred to the other, or the two are indifferent.

b. Transitivity - Both the preference and indifference relations for lotteries are transitive; i. e., given three lotteries A, B, and C, if A is preferred (indifferent) to B, and B is preferred (indifferent) to C, then A is preferred (indifferent) to C.

c. Reducibility - In case a lottery has as one of its alternatives another lottery, then the first lottery is decomposable into the more basic alternatives through the use of the probability calculus. An example is as follows: Consider a lottery of tossing a fair coin. The coin is tossed a second time, if it turned up tail on the first toss. The consequences are shown below:



An equivalent simple lottery is as follows:



d. Substitutibility - In any compound lottery, lotteries are interchangeable if they are equivalent.

e. Monotonicity - If two lotteries involve the same two alternatives, then the one in which the more preferred alternative has a higher probability of occuring is itself preferred.

f. Continuity - If A is preferred to B and B to C, then there exists a lottery involving A and C (with appropriate probabilities) which is indifferent to B. An example (Lifson, 1971) involves a $3 lottery ticket which pays off $10 or wins nothing. The player has the choice of either purchasing the ticket or not playing. Whether the player decides to play or not is dependent upon his preference. To quantify his preference as to the $3, we consider a simple lottery where 100 slips of papers numbered from 1 through 100 are placed in a hat. The following table tallies the player's preference:

Win $10 if No. Drawn is in the Set	Win Nothing if No. Drawn is in the Set	Probability of Win	Player's Preference
1	2-100	.01	$3
1-2	3-100	.02	$3
1-3	4-100	.03	$3
1-4	5-100	.04	$3
.	.	.	.
.	.	.	.
.	.	.	.
1-97	98-100	.97	Lottery
1-98	99-100	.98	Lottery
1-99	100	.99	Lottery
1-100		1.00	Lottery

There is some point in the table that the player is indifferent as to the $3 or the lottery.

Let A be the outcome of wining $10,

B be the outcome of keeping $3,

and C be the outcome of losing $3.

Outcome A is preferred to B, and B to C. The preceding lottery involving A and C, plus the probability associated with the indifference point, is what the continuity property means.

Basically, there are two types of simple lotteries. Given outcome h_1, h_2, and h_3 such that

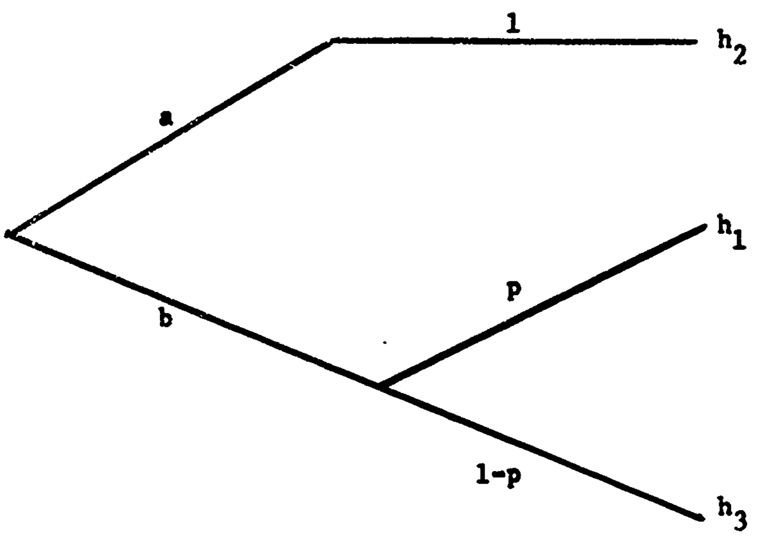
$$h_1 \geq h_2 \geq h_3$$

A decision maker is faced with the following choices:

Choice a: h_2 with certainty

Choice b: $(p)h_1$, $(1-p)\ h_3$)

In diagramatic form, the choices are as shown below:



Lottery 1: The decision maker is to establish p so that he is indifferent as to choices a and b. The example presented on page 9 is this kind of lottery.

Lottery 2: The decision maker is to establish h_2 so that he is indifferent as to choices a and b. An example of this kind of lottery is shown next.

Utility Function Determination

Prof. Ralph Swalm of Syracuse University (1971) designed a simple experiment to determine an individual's utility function. On page 13 is a form which facilitates the utility function determination. An individual is asked to choose between an alternative which leads to a certain gain (or loss) of a known amount and another alternative which could lead to either of two outcomes. It is assumed that all income or losses will take place in the very near future, with all amounts considered after taxes. The individual is acting in the capacity of a decision making of corporate/office funds, not private funds. Also, the responses should represent the actual action one would take if the alternatives are presented today, not what one feels he should do, not what might be expected to be done.

Conceptually, the questions asked are of the following form:

Suppose two alternatives are posed. The first involves undertaking to bid on a new project. If the bid is successful, the office makes a net profit of, say, $100,000. If unsuccessful, the costs for making the bid is reimbursed, but the net gain is zero. The best available information leads to assign a 50-50 chance to the above events.

The second alternative is to put the manpower, instead of making the bid, into cost reduction efforts Based on past experience, it is certain that this would result in a net gain. How large would this gain have to be to make one indifferent as to which alternative to take? In other words, at what certain income would one be indifferen to the office s gain or getting a 50-50 chance of making $100,000 or nothing?

Other questions can be constructed such as the following.

Suppose the company is being sued for x dollars, and the probability of losing is 50%. What amount of x would you be willing to pay to settle out of court?

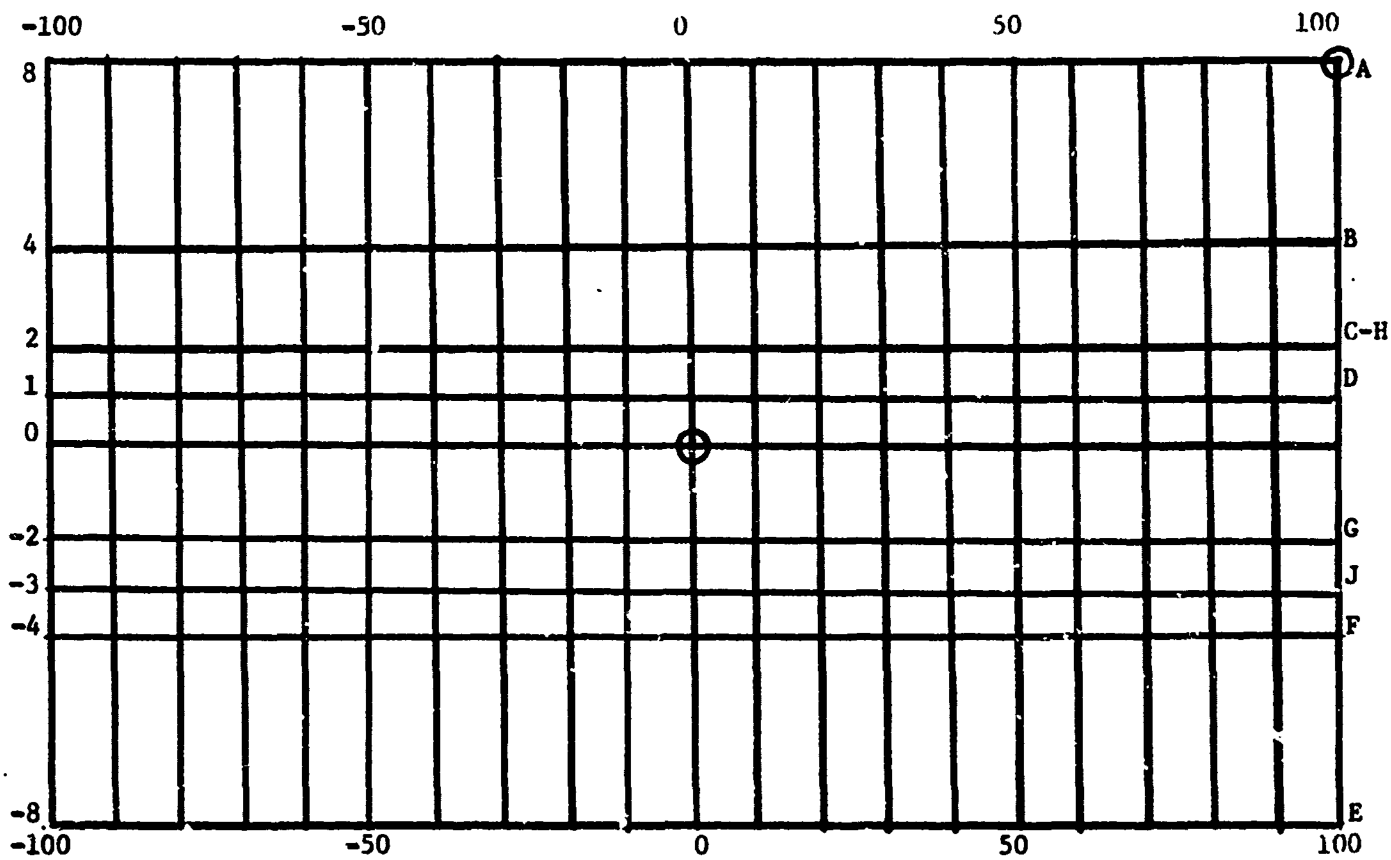
Automated lotteries are available where the decision maker can converse with a computer and/or graphic unit to establish the utility functions. One system is called "Lexicoder" of Lex Computer Systems, Inc., Palo Alto, California.

UTILITY FUNCTION DETERMINATION

1. A________ or Zero vs B________
2. B________ or Zero vs C________
3. C________ or Zero vs D________
4. A________ or E________ vs Zero
5. E________ or Zero vs F________
6. F________ or Zero vs G________
7. A________ or F________ vs F________
8. C________ or E________ vs J________

	AMOUNT	%A	UTILES
A			+8
B			+4
C			+2
D			+1
E			-8
F			-4
G			-2
H			2
J			-3

% of Planning Horizon



III. AN EXAMPLE

Utility concepts have been applied to industrial problems (Matheson, 1969; Spetzler, 1968; Howard, 1966). There is a lack of case studies in the Defense Department. The example presented below is intended to illustrate some key features of utility theory applied to the defense acquisition process. It is a hypothetical example constructed by the author who is solely responsible for the accuracy of the contents. The approach is similar to an example by Kaufman (1970).

The example is concerned with the air-armament of two aircraft denoted by A1 and A2. Two gun candidates are available to fulfill the air-armament role; these two gun candidates are denoted by G1 and G2. On performance and effectiveness alone, G2 is superior to G1. Aircraft A1 can accept either gun. Aircraft A2 can accept G1; should G2 be adopted for it, a major redesign is required in the gun turret, as well as the aircraft structure. On the other hand, G1 is more readily available than G2 and is less costly. The objective is to evaluate the two guns with respect to cost, time, performance, and risk so as to determine the most suitable gun system for application to the aircraft.

With two guns and two aircraft, there are four possible combinations. We identify these four combinations as four distinct performance levels shown below.

Performance Level	Guns Applied to Aircraft A1	Guns Applied to Aircraft A2
1	G1	G2
2	G1	G1
3	G2	G2
4	G2	G1

From a qualitative standpoint, an overall comparative analysis can be presented in a matrix shown below:

EVALUATION DIMENSION	AIRCRAFT A1 G1	AIRCRAFT A1 G2	AIRCRAFT A2 G1	AIRCRAFT A2 G2
Cost	Good	Good	Good	Fair
Time	Good	Fair	Good	Poor
Effectiveness	Good	Very Good	Good	Good
Risk	Low	Low	Low	Medium

Although this matrix reveals the qualitative comparison among the various contenders with respect to the evaluation dimensions, it does not facilitate a quantitative trade-off for decision-making. We will now proceed with the decision/risk analysis through a cost-sensitivity analysis, a simplified decision model, and a refined decision model to effect a quantitative comparative analysis.

Cost Model

From a cost analysis with supporting cost rationale, a cost model is constructed by regression analysis. This model relates total life-cycle costs of guns to quantities of aircraft armed with the guns and also of mixes of two guns on two aircraft.

Suppose n_α represents the quantity of α-aircraft, and c_α^β represents total life-cycle cost of β-gun applied to α-aircraft. The equations for costs in \$-millions are as follows:

$$c_{A1}^{G1}(n_{A1}) = 0.1343\, n_{A1} + 15.6,$$

$$c_{A1}^{G2}(n_{A1}) = 0.0837\, n_{A1} + 17.3,$$

$$c_{A2}^{G1}(n_{A2}) = 0.1343\, n_{A2} + 15.6,$$

$$c_{A2}^{G2}(n_{A2}) = 0.0837\, n_{A2} + 10.8 + c_t,$$

where c_t represents the turret development cost for G2 applied to A2.

For the four combinations of performance levels, total life-cycle costs of guns in \$-millions for arming two aircraft are tabulated below.

Performance Level	Total Cost		Cost to Arm A1		Cost to Arm A2	Duplicating Fixed Cost	Modification Cost
1	c_1	=	$c_{A1}^{G2}\ (n_{A1})$	+	$c_{A2}^{G1}\ (n_{A2})$		
2	c_2	=	$c_{A1}^{G1}\ (n_{A1})$	+	$c_{A2}^{G1}\ (n_{A2})$	− 5.0	
3	c_3	=	$c_{A1}^{G2}\ (n_{A1})$	+	$c_{A2}^{G2}\ (n_{A2})$	− 6.9	$+ c_m$
4	c_4	=	$c_{A1}^{G1}\ (n_{A1})$	+	$c_{A2}^{G2}\ (n_{A2})$		$+ c_m$

where c_m is the modification cost which has not been estimated accurately. For performance levels 2 and 3, the same gun is applied to both aircraft; hence, a duplicating fixed cost must be substracted from the total cost.

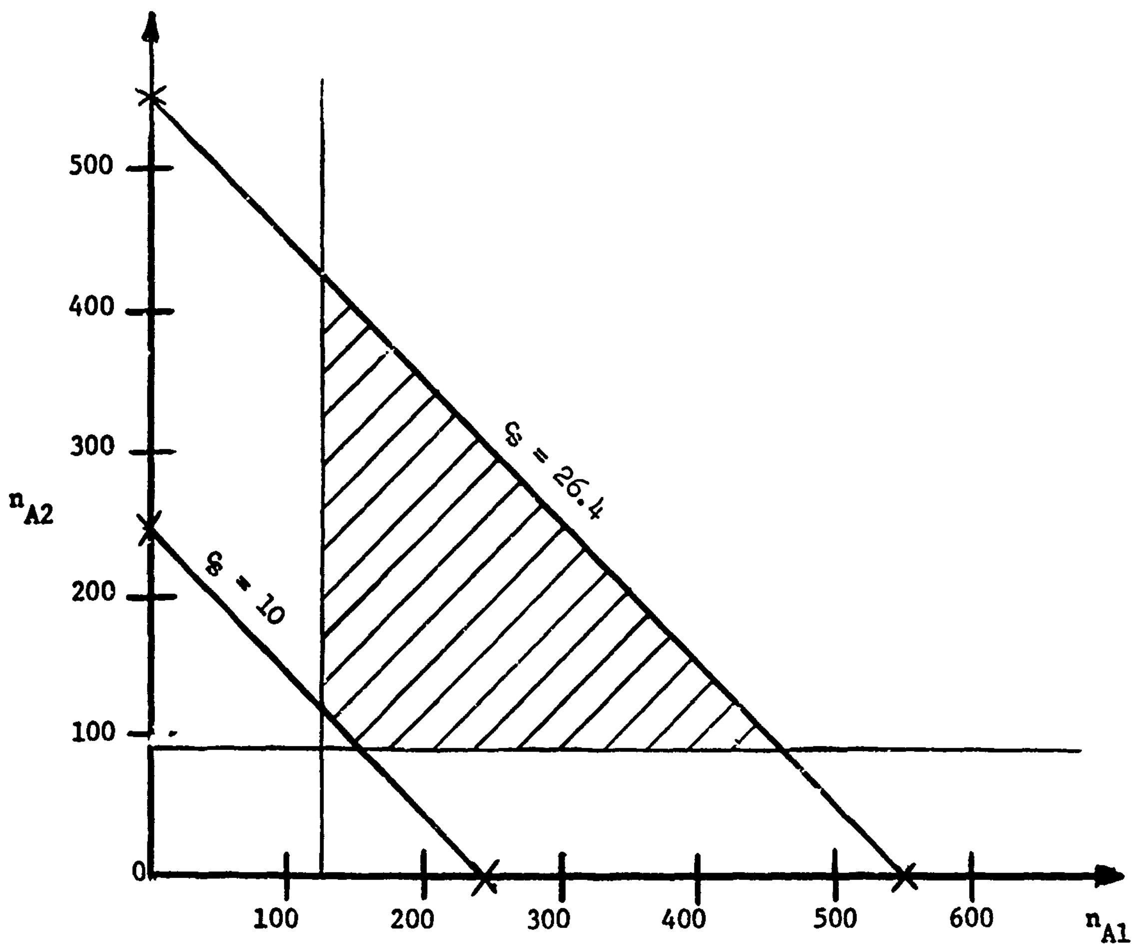
From the above, cost sensitivity can be checked on c_t and c_m to derive some criteria which are critical to the decision. Assuming cost dominance for the above four mixed life-cycle costs, we can easily derive some criteria. The derivation of these criteria is simple; we begin with the cost dominance, substitute the cost equations for the dominant behavior, solve for the quantities n_{A1} and n_{A2}, and arrive at the criteria shown on the next page.

Cost Dominance	Criteria $(c_s \equiv c_m + c_t)$
$c_1 \leq c_2$	1. $n_{A1} \geq 130$
$c_2 \leq c_3$	2. $n_{A1} + n_{A2} \leq 25 + 20c_s$
$c_1 \leq c_3$	3. $n_{A2} \leq 20c_s - 110$
$c_3 \leq c_4$	4. always true
$c_2 \leq c_4$	5. $n_{A2} \leq 130 + 20c_s$
$c_1 \leq c_4$	6. $n_{A2} \leq n_{A1} + 20c_s$

Criteria 4, 5, and 6 are not as stringent as criteria 1, 2, and 3. As we like to have the above cost dominance behavior, we search for aircraft quantity mixes which satisfy the first three criteria, and we can ignore the criteria 4, 5, and 6. This is done by plotting n_{A1} versus n_{A2} under the three constraints, plus the following estimate for c_s:

$$10 \leq c_s \leq 26.4 .$$

The shaded area in the following diagram represents all feasible combinations of aircraft satisfying the three constraints.

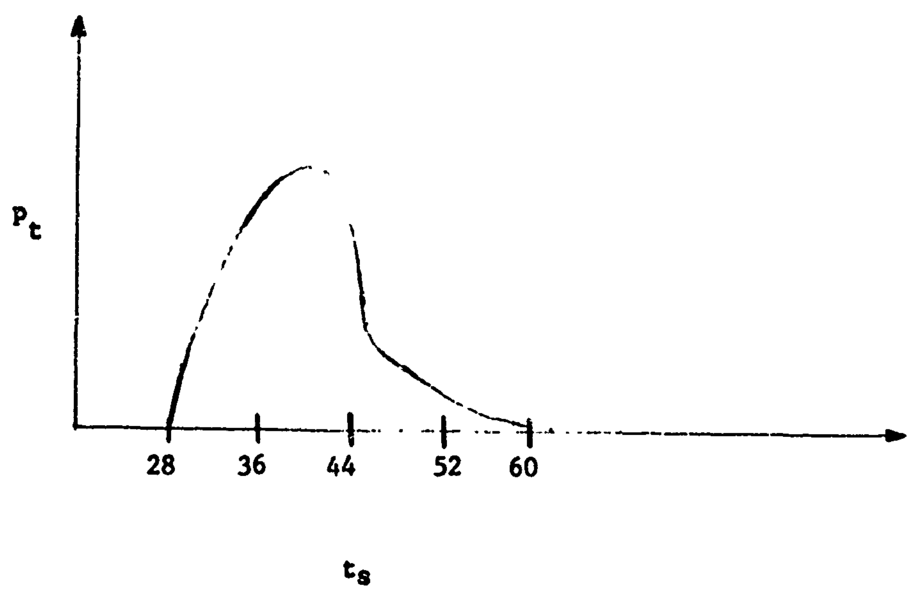


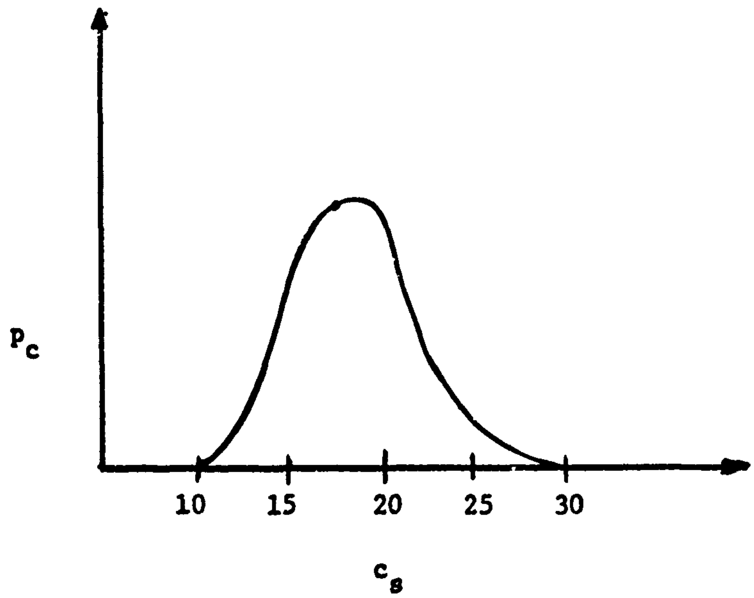
Decision Model

Acquisition times in months up to the end of Engineering Test/Service Test (ET/ST) for the respective guns with the aircraft are as follows.

Gun	Aircraft A1	Aircraft A2
G1	19	19
G2	22	28-60

Since the turret development and aircraft modification are uncertain for G2 applied to A2, the cost c_s and time t_s are assumed as two random variables. Subjective probability density function p are solicited. Let the probability density functions be of the following shape:





Based on these probability density functions, we can calculate the expected values for c_s and t_s, denoted by $E(c_s)$ and $E(t_s)$, respectively. Since p_c is symmetric about 18.2,

$$E(c_s) = 18.2 .$$

The density function is fitted by a beta density, i.e.,

$$f(x) = \frac{\Gamma(p+q)}{\Gamma(p)\Gamma(q)} x^{p-1}(1-x)^{q-1} , \quad 0 \leq x \leq 1, \quad p,q > 1.$$

Setting $p=2$ and $q=4$, we obtain

$$p_t(x) = 20x(1-x)^3 ,$$

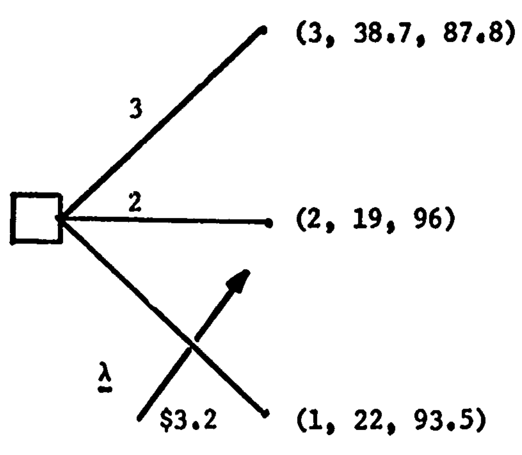
where $t_s = 32x + 28$, and

$$E(t_s) = \int_0^\infty t_s \, p_t(t_s) dt_s = 38.7.$$

At this time, we shall apply the utility concepts to resolve the problem. Assuming the reader is not totally familiar with the utility scheme, we approach the decision analysis in two parts: first, a simplified version; secondly, a more complete analysis. Both parts are concerned with one feasible combination consisting of 575 aircraft with 375 A2 and 200 A1. This combination is within the cost dominance solution set in the cost model.

Simplified Model

The basic decision problem at hand is as show below in the diagram. It is assumed that any one of the three performance levels is achievable.



The $3.2 million is the toll for developing both guns concurrently, for parallel development requires an additional initial $3.2 allotment of R&D funds. If the $3.2 million is not allotted initially, then only the performance levels 2 and 3 can be pursued. If $3.2 million can be funded, then this $3.2 is included in the total life-cycle cost for that performance level.

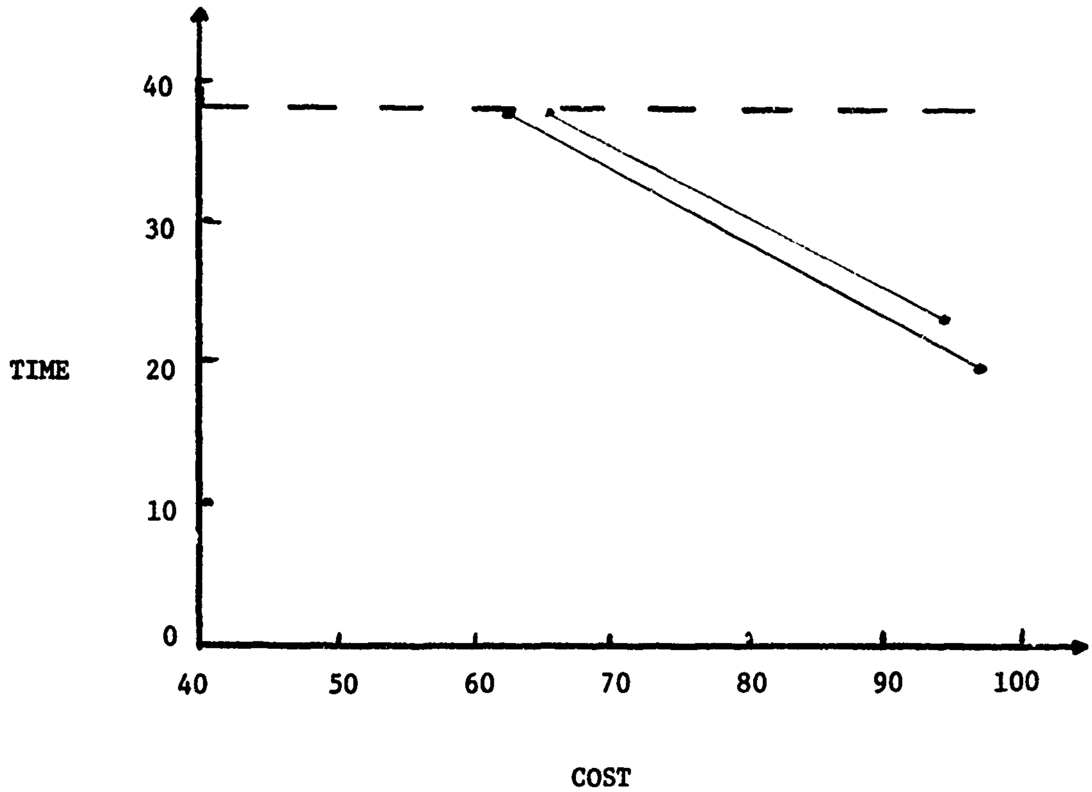
The triples for the three levels represent the performance level, acquisition time, and total cost for the three outcomes. Performance level 4 is not in the decision tree, as all three performance levels dominate that level, provided t_s and c_s are close to the expected values.

To facilitate utility assessment, we first reduce the dimensions in the triples by eliminating the time dimension so that all times are on one equivalent time basis. It is also assumed that there is no time constraint; otherwise, the constraint would reduce the number of possible routes. To accomplish this reduction process, we construct the so-called time-cost indifferences curves by posing preference questions to the decision-makers as follows:

For performance level 2, do you profer time-cost pair 1 or pair 2?

Time-cost Pair 1	Time-cost Pair 2	Choice
(19,96)	(38.7,90)	1
(19,96)	(38.7,75)	1
(19,96)	(38.7,50)	2
(19,96)	(38.7,60)	2
(19,96)	(38.7,65)	1
(19,96)	(38.7,62)	indifferent

In this fashion, time-cost indifference curves can be generated as shown in the following plot.



Thus, we have

(19,96) is indifferent to (38.7,62)

and (22,93.5) is indifferent to (38.7,64).

On the basis of 38.7 months as the reference time, the outcomes involve only performance and cost:

(1,64)

(2,62)

(3,87.8)

To assess the utilities of these three outcomes, we enumerate some additional outcomes so as to establish the scale references; the outcome (1,62) is scaled to 1-utile, and (2,87.8) is scaled to 0-utile. All other outcomes would be in the interval between 0-utile and 1-utile. To determine the utilities of the intermediate outcomes such as (2,62), we apply a lottery to obtain $\alpha = u(2,62)$, so that

(2,62) is indifferent to

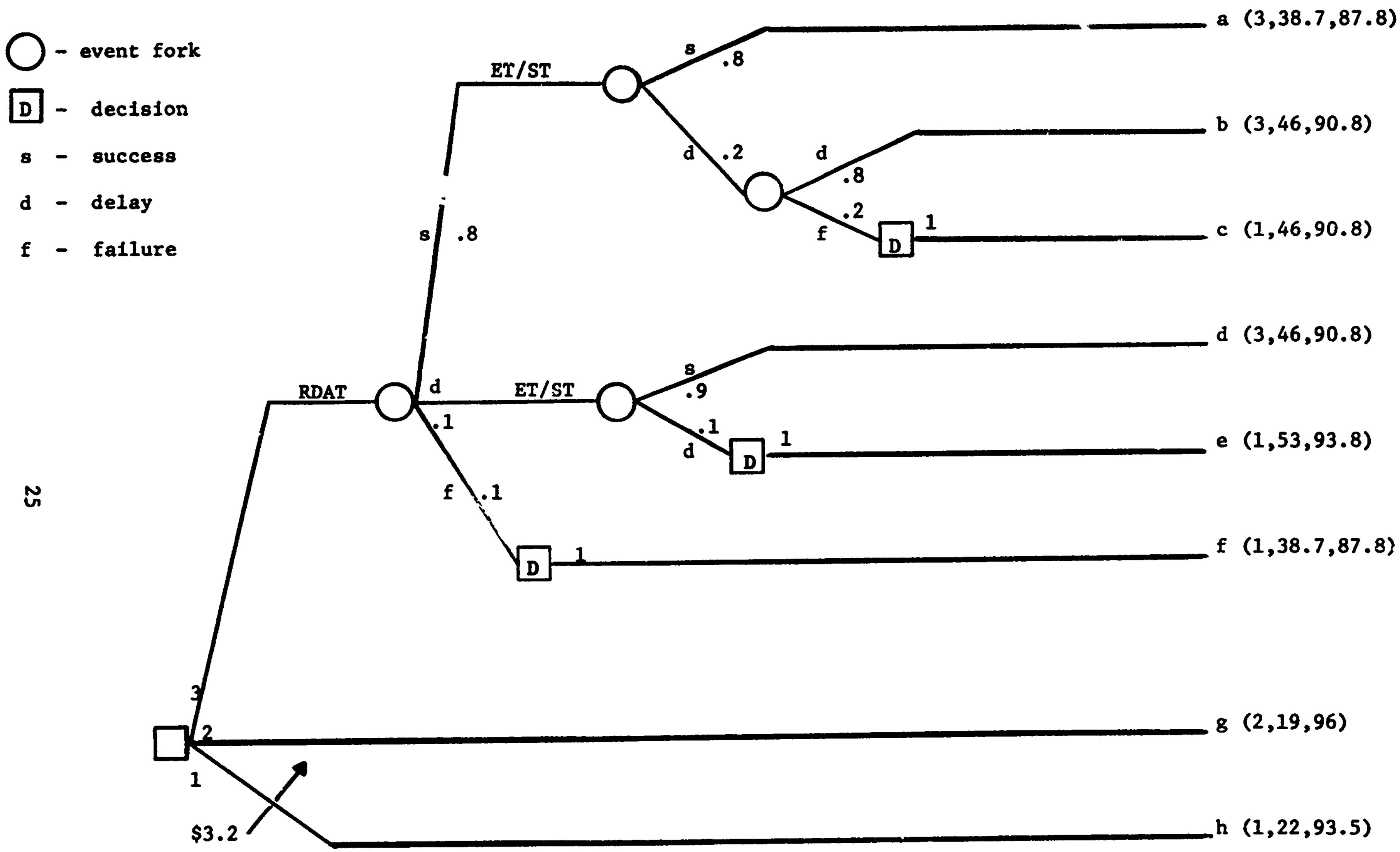
After a series of lottery, we would have the utilities of all the outcomes as follows.

Outcome (p,c)	u(p,c) in utiles
(1,62)	1.00
(1,64)	.95
(2,62)	.75
(3,87.8)	.50
(2,87.8)	0.00

Hence, the decision problem has been quantified with the utilities of pursuing performance levels 1, 2, and 3, being .95, .75, and .5, respectively.

Refined Model

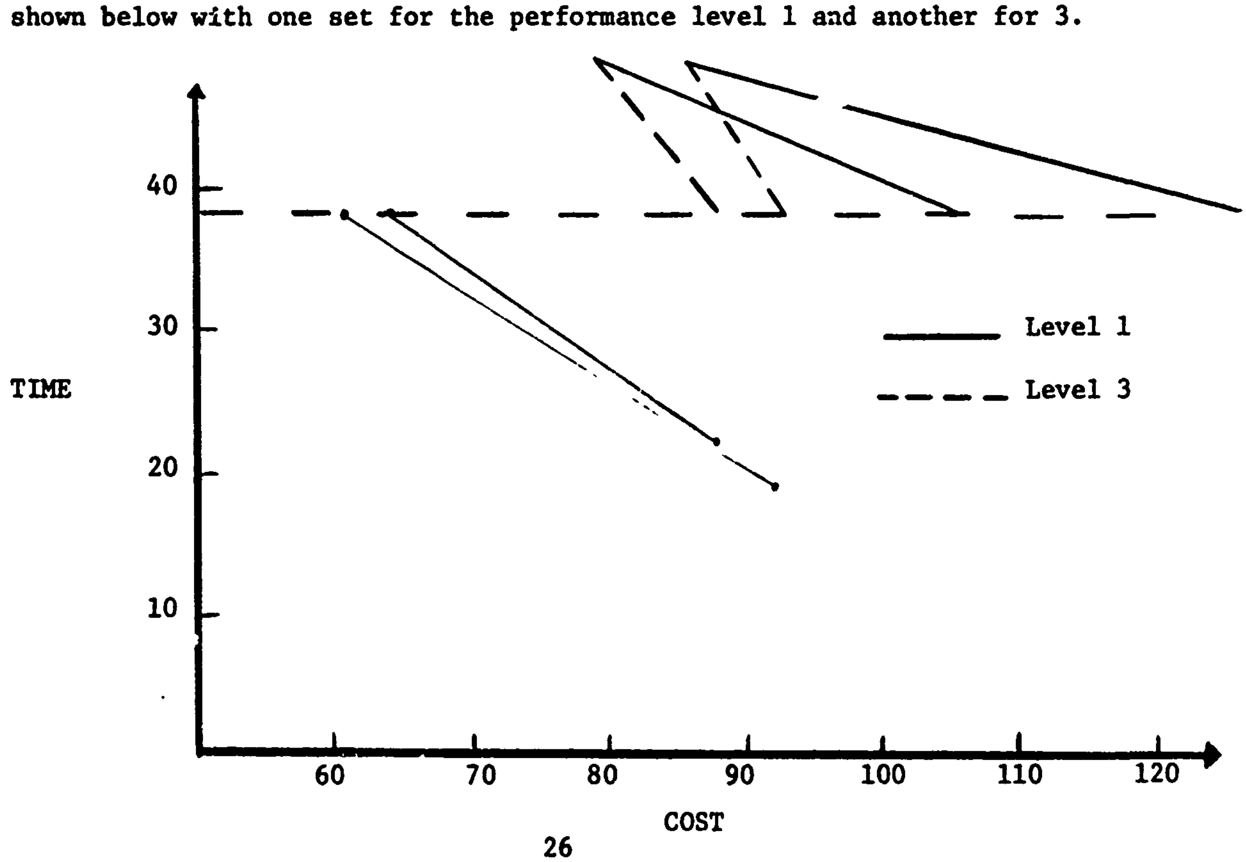
Now, the analysis is carried out a second time to include major critical milestones. From the technical risk analysis, information is gathered so as to interface with the decision analysis. Specifically, careful assessment of the problems, consequences of failure, and judgment of effort needed for a practical solution highlighted such problems for the G2 applied to A2 as projectile cook-off, maintainability, recoil and blast, aluminum cartridge case, G-load, and structural modifications. Key milestones include Research & Development Acceptance Test (RDAT) and ET/ST. The next page shows a decision-tree reflecting the major program milestones. Risk analysis contributed



DECISION-TREE

to the establishment of the probabilities of occurrences at each event fork. Outcome triples of performance level, time, and cost are also shown at the terminal nodes. It is noted that if the development is completely successful, we have an outcome a. If it is completely unsuccessful, then outcome f results, with G1 as the back-up gun system for A2. If minor problems are encountered at RDAT or ET/ST, then a delay occurs which calls for additional time and resources; timely resolution would yield outcomes b, d, and e. Otherwise a decision for applying the G1 to A2 results, and outcomes c and e are realized. Outcomes g and h are simply the other two approaches.

Again, similar to the technique in the simplified model, we begin by the reduction of the triples to only two dimensions by eliminating the time dimension. Two sets of time-cost indifferences curves are shown below with one set for the performance level 1 and another for 3.



By using the time-cost indifference curves, we ascertain the equivalent outcome set shown below, based on 38.7-month reference time frame.

Outcomes	Equivalent Outcomes
a (3,38.7,87.8)	(3,87.8)
b (3,46,90.8)	(3,96)
c (1,46,90.8)	(1,105)
d (3,46,90.8)	(3,96)
e (1,53,93.8)	(1,125)
f (1,38.7,87.8)	(1,87.8)
g (2,19,96)	(2,62)
h (1,22,93.5)	(1,64)

Next, we assess the utilities of these outcomes and apply the concept of "averaging out and folding back" (Raiffa, 1968).

Let the utility function of performance level p and cost c be in terms of the best possible outcome and worst outcome.

$$u(p,c) = v(p)u(1,c) + (1 - v(p))u(2,c).$$

Then $$u(1,c) = x(c)u(1,c^*) + (1 - x(c))u(1,c_*),$$

and $$u(2,c) = y(c)u(2,c^*) + (1 - y(c))u(2,c_*),$$

where c^* is the lowest cost, and c_* is the highest cost, namely 62 and 125, respectively. To obtain these utility functions, it is necessary to establish four sets of lotteries.

1. $u(1,c)$: $\beta = x(c)$

$(1,c)$ is indifferent to $\left\langle \begin{array}{ll} \beta & (1,c^*) \\ 1-\beta & (1,c_*) \end{array} \right.$

2. $u(2,c)$: $\gamma = y(c)$

$(2,c)$ is indifferent to $\left\langle \begin{array}{ll} \gamma & (2,c^*) \\ 1-\gamma & (2,c_*) \end{array} \right.$

3. $u(p,c)$: $\delta = v(p)$

(p,c) is indifferent to $\left\langle \begin{array}{ll} \delta & (1,c) \\ 1-\delta & (2,c) \end{array} \right.$

4. By scaling $u(1,c^*) = 1$ and $u(2,c_*) = 0$, two lotteries are needed to establish $u(1,c_*)$ and $u(2,c^*)$:

$(1,c_*)$ is indifferent to $\left\langle \begin{array}{ll} \varepsilon & (1,c^*) \\ 1-\varepsilon & (2,c_*) \end{array} \right.$

and a similar lottery for $u(2,c^*)$.

The results of the first two sets of lotteries are tabulated below:

c	$x(c)$	$y(c)$
62	1.0	1.0
64	.95	.97
87.8	.60	.65
96	.30	.35
105	.10	.15
125	0.0	0.0

The third set of lotteries yields simply the following:

p	$v(p)$
1	1.0
2	0.0
3	0.5

Lastly, the fourth set of lotteries results in

$$u(1,c_*) = .1 \quad \text{and} \quad u(2,c^*) = 0.75.$$

These utilities are now combined to find u(p,c) by the utility equations and the lottery outcomes:

c	u(1,c)	u(2,c)	u(3,c)
62	1.0	.75	.875
64	.955	.7275	.841
87.8	.64	.4875	.564
96	.37	.2625	.316
105	.19	.1125	.151
125	0.0	0.0	0.0

These utilities are used to find the utilities of the outcomes.

Outcomes	Utilities
a	.564
b	.316
c	.19
d	.316
e	0.0
f	.64
g	.75
h	.955

To fold back, we start from the terminal nodes, multiply the utilities at terminal nodes with the respective probabilities of occurrence, and sum at each event fork. Utilities of the event forks are folded back by again multiplying the utilities with the respective probabilities of occurrences and summing. This process is repeated to the initial decision node, and we can find the expected utilities of the three performance levels. In similar fashion, we can fold back the costs and times.

Performance Level	Expected Cost	Expected Time	Utilities
1	93.5	22	0.96
2	96	19	0.75
3	88.8	40.7	0.5

At this point, the "averaging out and folding back" technique shows that the expected utility for performance level is highest; the second level second best; and the third level third. The decision-maker established a quantitative comparison among the three alternatives with betting odds of 96 to 75 to 50 for performance levels 1, 2, and 3, respectively. A qualitative comparative analysis has been transformed to a quantitative one, and a decision-maker should pursue according to the preference pattern above.

IV. CONCLUSION

In this paper, it is emphasized that risk analysis must interface with decision analysis to facilitate decision-making for major developmental programs in the materiel acquisition process. Risk analysis contributes to uncertainties resolution, decision-tree structuring, and probabilities of occurrences of major program events through the assessment of problems, consequences of failure, and judgment as to effort needed for a practical solution. The utility concept plays a significant role in the quantification of preferences and subjective judgment. A risk analysis is complete only if these two areas are properly tied together.

At this time, a complete, well-documented, real-life case study of a major developmental program is still very much needed to bridge the credibility gap.

APPENDIX

Utility Axioms

Consider a finite exhaustive set of mutually exclusive outcomes h_i, $i \varepsilon N$, where N is some initial segment set $(i = 1, 2, \ldots, n)$. The set of outcomes can be denoted by an n-tuple:

$$(h_1, h_2, \ldots, h_n) = H.$$

Also, for $i \neq j$, $i,j \varepsilon N$,

$$h_i \cap h_j = 0.$$

Suppose for each outcome h_i, a probability p_i is known to exist such that for each i, $i \varepsilon N$,

i. $0 \leq p_i \leq 1$,

ii. $\sum_{i=1}^{n} p_i = 1$.

and

iii. Probability $(h_i \cup h_j) = p_i + p_j$,

$i \neq j$, $i,j \varepsilon N$ the set of probability can also be denoted by a corresponding n-tuple:

$$(p_1, p_2, \ldots, p_n) = P.$$

A lottery is a chance mechanism which yields H with the known P over N.

Notation: A lottery L is denoted as an n-tuple:

$$L = (p_1h_1, p_2h_2, \ldots, p_nh_n),$$

where one and only one outcome h_i will be realized and the probability that it will be h_i is p_i.

The concept of preference is prescribed by the relations defined below.

Definition 1: In $i \neq j$, $i,j \in N$,

i. $h_i \geq h_j$ denotes h_i is at least as preferred as h_j.

ii. $h_i > h_j$ denotes h_i is preferred to h_j.

iii. $h_i \sim h_j$ denotes h_i is equivalent (or indifferent) to h_j.

Axiom 1: (Comparability) For i,j N, $i \neq j$, only one of the following holds:

i. $h_i \geq h_j$

ii. $h_j \geq h_i$

Both relations are true simultaneously provided $h_i \sim h_j$.

Notation: From here on, H will be arranged such that

$$h_1 \geq h_2 \geq h_3 \geq \cdots \geq h_n.$$

Axiom 2: (Transitivity) If $h_i \geq h_j$, and $h_j \geq h_k$, $i,j,k \in N$, then

$$h_i \geq h_k$$

Axiom 3: (Transitivity) Preference and indifference among lotteries are transitive relations.

Axiom 4: (Continuity) Suppose $h_1 \geq h_i \geq h_n$, i N. There exists a real number u_i, $0 \leq u_i \leq 1$ such that

$$h_i \sim (u_i h_1, (1-u_i) h_n).$$

Axiom 5: (Substitutability) If $h_i \sim h_i^*$, then

$$(p_1 h_1,\ p_2 h_2,\ \cdots,\ p_i h_i,\ \cdots,\ p_n h_n)$$
$$\sim (p_1 h_1,\ p_2 h_2,\ \cdots,\ p_i h_i^*,\ \cdots,\ p_n h_n).$$

Axiom 6: (Reducibility) A compound lottery can be reduced to an equivalent simple lottery with outcomes $h_1, h_2, \cdots, h_n$, their probabilities computed according to ordinary probability calculus.

Theorem: A lottery L can be transformed into another lottery L' such that

a. L' involves only h_1 and h_n.

b. L' $\sim$ L.

Proof:

By the continuity axiom, there is u_i for each h_i such that

$$h_i \sim (u_i h_1, (1-u_i) h_n).$$

By substitution, each h_i can be replaced accordingly:

$$\begin{aligned} L &= (p_1 h_1,\ p_2 h_2,\ \cdots,\ p_i h_i,\ \cdots,\ p_n h_n) \\ &\sim ((p_1 u_1 h_1,\ p_1(1-u_1) h_n),\ (p_2 u_2 h_1,\ p_2(1-u_2) h_n), \\ &\quad \cdots,\ (p_i u_i h_1,\ p_i(1-u_i) h_n),\ \cdots, \\ &\quad (p_n u_n h_1,\ p_n(1-u_n) h_n)) \end{aligned}$$

By reduction of compound lottery axiom, to only h_1 and h_n,

$$L' = ((p_1u_1 + p_2h_2 + \cdots + p_iu_i + \cdots + p_nu_n)h_1,$$

$$(p_1(1-u_1) + p_2(1-u_2) + \cdots + p_i(1-u_i) + \cdots + p_n(1-u_n))h_n)$$

$$= (h_1 \sum_{i=1}^{n} p_iu_i, h_n(1 - \sum_{i=1}^{n} p_iu_i)) \sim L.$$

<u>Axiom 7</u>: (Monotonicity) Consider two lotteries

$$L_p = (ph_1, (1-p)h_n) \text{ and } L_q = (qh_1, (1-q)h_n).$$

$$L_p \gtrsim L_q \text{ if and only if } p \geq q.$$

<u>Definition</u>: A utility of an outcome h_i in a lottery L is a measure u_i such that the preference or indifference relationship satisfies the above axioms.

For two lotteries L_p and L_q such that utility measures $\{u_i\}$ associated with the basic outcomes have been determined, the preferences for the lotteries are reflected by the following:

$$p = \sum_{i=1}^{n} p_iu_i$$

and

$$p = \sum_{i=1}^{n} q_iu_i,$$

where p and q are called expected utilities of the lotteries L_p and L_q, respectively.

<u>Theorem</u>: A linear transformation does not affect the relative preference rankings among lotteries.

Proof: Let $u_i^* = au_i+b$ be the linear transformation, where a and b are constants, $a \neq 0$, $i \varepsilon N$. Then for the j-th lottery,

$$E(u_i^*)_j = \sum_{i=1}^{n} p_i^j \,(au_i+b)$$

$$= a \sum_{i=1}^{n} p_i^j \, u_i + b \sum_{i=1}^{n} p_i^j$$

$$= a \; E(u_i)_j + b.$$

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